From research questions to statistics

Lecture 2

Dr Milan Valášek
31 January 2022

UNIVERSITY OF SUSSEX

Today

- Point estimates vs interval estimates
- Confidence intervals
- *t*-distribution

What stats is about (yet again)

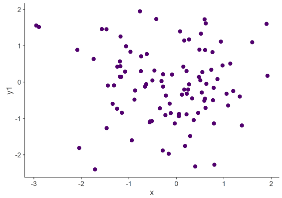
- We want to know about the world (population)
- We can only get data from samples
- We calculate statistics on samples and use them to *estimate* the values in population
- Statistics is all about *making inferences about populations based on samples*
- If we could measure the entire population, we wouldn't need stats!

Point estimates

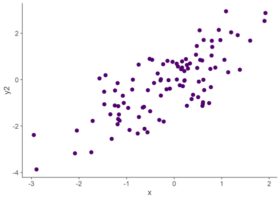
- You've heard of the sample mean, median, mode
- These are all point estimates - single numbers that are our best guesses about corresponding *population parameters*
- Measures of spread (*SD*, σ^2, *etc.*) are also point estimates
- Even relationships between variables can be expressed using point estimates

Point estimates

r = −.07



r = .752



Accuracy and uncertainty

- Sample mean $\bar{x}$ is the best estimate $\hat{\mu}$ of population mean but means of almost all samples differ from population mean μ
- Same is true for *any* point estimate
- *SE* of the *mean* expresses the uncertainty about the estimates of the population *mean*
- *SE* can be calculated for other point estimates, not just the mean
- We can quantify uncertainty around point estimates using **interval estimates**

Interval estimates

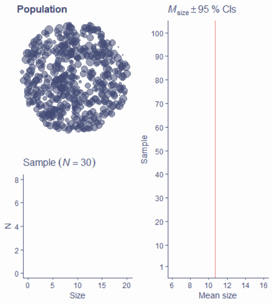
- In addition to estimating a single value, we can also estimate an interval around it
- *e.g.,* mean = 4.13 with an interval from −0.2 to 8.46
- Interval estimates communicate the uncertainty around point estimates
- There are different kinds of interval estimates
    - Important: **confidence intervals**

Confidence interval

- We can use *SE* and the sampling distribution to calculate a confidence interval (CI) with a certain *coverage*, *e.g.,* 90%, 95%, 99%...
- For a 95% CI, 95% of these intervals around sample estimates will contain the value of the population parameter
- Let's see an example

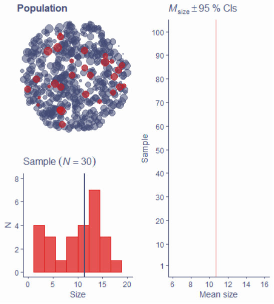
Confidence interval

- Population of circles of different sizes



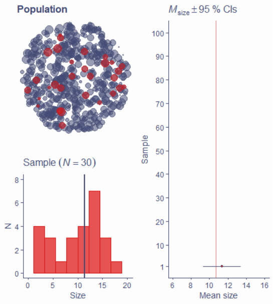
Confidence interval

- Sample from population, estimate mean size



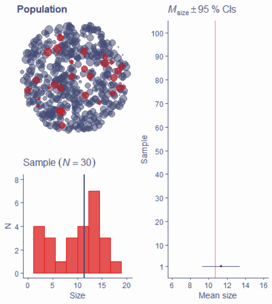
Confidence interval

- Calculate the 95% CI around the mean



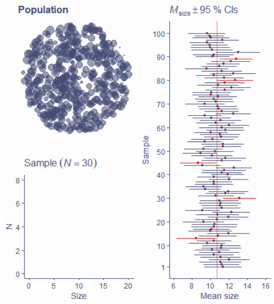
Confidence interval

- Lather, rinse, repeat...



Confidence interval

- ~5% don't contain population mean = 95% coverage

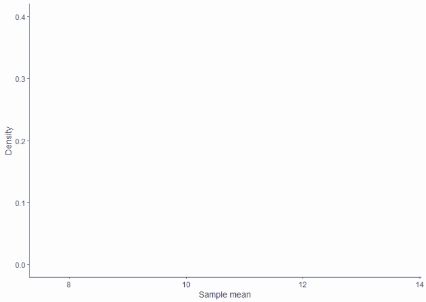


How is it made?

- Easy if we know sampling distribution of the mean
- 95% of sampling distribution is within ±1.96 *SE*
- 95% CI around estimated population mean is mean ±1.96 *SE*

How is it made?

- Sampling distribution of the mean is normal (as per CLT)
- Middle 95% of the sample means lie within ±1.96 *SE*
- We use the same 1.96 *SE* to construct 95% CI around mean

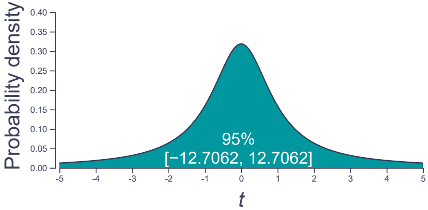


How is it made?

- Sampling distribution is, however, not known!
- It can be approximated using the *t*-distribution and *s* and *N*

t-distribution

- Symmetrical, centred around 0
- Its shape changes based on **degrees of freedom**
- As shape changes, so do proportions (unlike with normal)
- In standard normal, middle 95% of data lie within ±1.96
- In *t*-distribution, this critical value changes based on *df*



df = 1

t-distribution

- *t*-distribution crops up in many situations
- Always has to do with **estimating sampling distribution from a finite sample**
- How we calculate number of *df* changes based on context
  - Often has to do with N, number of estimated parameters, or both
  - In the case of sampling distribution of the mean, $df = N - 1$

Back to CI

- 95% CI around estimated population mean is mean ±1.96 *SE* **if we know the exact shape of sampling distribution**
    - We don't know the shape so we approximate it using the *t*-distribution
- We need to replace the 1.96 with the appropriate critical value for a given number of *df*
- For *N* = 30, $t_{crit}(df=29) = 2.05$

```
qt(p = 0.975, df = 29)
```

```
## [1] 2.04523
```

Back to CI

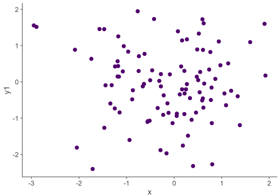
- 95% CI around the mean for a sample of 30 is $\bar{x} \pm 2.05 \times SE$
- $\widehat{SE} = \frac{s}{\sqrt{N}}$
- $95\% \ CI = Mean \pm 2.05 \times \frac{s}{\sqrt{N}}$
- To construct a 95% CI around our estimated mean, all we need is
  - Estimated mean (*i.e.* sample mean, because $\hat{\mu} = \bar{x}$)
  - Sample *SD* (*s*)
  - *N*
  - Critical value for a *t*-distribution with *N* − 1 *df*

CIs are useful

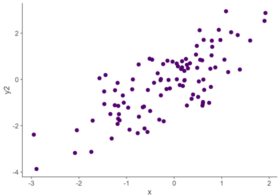
- Width of the interval tells us about how much we can expect the mean of a different sample of the same size to vary from the one we got
- There's a x% chance that any given x% CI contains the true population mean
- **CAVEAT:** That's not the same as saying that there's a x% chance that the population mean lies within our x% CI!
- CIs can be calculated for *any point estimate*, not just the mean!

Remember this?

$r = -.07$

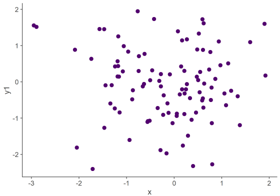


$r = .752$

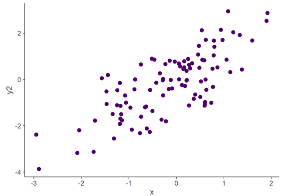


Remember this?

r = −.07; 95% CI [−.263, .128]



r = .752; 95% CI [.652, .827]



Take-home message

- Our aim is to *estimate unknown population characteristics* based on samples
- *Point estimate* is the best guess about a given population characteristic (parameter)
- Estimation is inherently *uncertain*
    - We cannot say with 100% certainty that our estimate is truly equal to the population parameter
- *Confidence intervals* express this uncertainty
    - The wider they are, the more uncertainty there is
    - They have arbitrary *coverage* (often 50%, 90%, 95%, 99%)
- CIs are constructed using the *sampling distribution*
    - True sampling distribution is unknown, we can approximate it using the *t*-distribution with given *degrees of freedom*
- CIs can be constructed for *any point estimate*
- For a 95% CI, there is a 95% chance that any given CI contains the true population parameter