From research questions to statistics

Lecture 3

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Today

- Conceptual, operational & statistical hypothesis
- Null Hypothesis Significance testing
- *p*-values

Hypothesis

- Statement about something in the world
  - Often in terms of differences or relationships between things/people/groups
- Must be testable: it must be possible for the data to either support or disconfirm a hypothesis
- Should be about a single thing

Levels of hypothesis

- *Conceptual*: Expressed in normal language on the level of concepts/constructs
- *Operational*: Restates a conceptual hypothesis in terms of how constructs are measured in a given study
- *Statistical*: Translates an operational hypothesis into language of mathematics

Conceptual hypotheses

- Expressed in normal language on the level of concepts/constructs
- **Good hypothesis:** *"The recent observed rising trend in global temperatures on Earth is primarily driven by human-produced greenhouse gas emissions."*
- **Bad hypothesis:** *"Homœopathic products can cure people, but sometimes they make them worse before they make them better, and the effect is only apparent subjectively with respect to some vague 'holistic' notions rather than a specific well-defined and testable set of criteria."*

From research question to conceptual hypothesis

- Let's say we're interested in factors predicting sport climbing performance
- *Research question:* Are there morphological characteristics that predispose some people to be better at climbing?
- We have a hunch that having relatively long arms might be beneficial
- *Conceptual hypothesis:* Climbers have relatively longer arms than non-climbers

Operationalisation

- To be able to formulate a hypothesis in statistical terms, we first need to get from the conceptual level to the level of measurement
- **Operationalisation** is the process of defining variables in terms of how they are measured
  - The *concept* of intelligence can be operationalised as total score on Raven's Progressive Matrices
  - The *concept* of cognitive inhibition can be operationalised as (some measure of) performance on the Stroop test.

Example: The Ape Index

- The ape index (AI) compares a person's arm span to their height
  - Positive AI means, that your arm span is larger then your height
  - 165 cm (5′5″) tall person with an arm span of 167 cm has an ape index of +2
  - Found to correlate with performance in some sports (*e.g.,* climbing, swimming, basketball)


Ashima Shiraishi
155 cm tall
Ape index +10 cm

Operational hypotheses

- *Conceptual hypothesis:* Climbers have relatively longer arms than non-climbers

- *Operational hypothesis:* Elite climbers have, on average, a higher ape index than general population

Statistical hypotheses

- Translation of an operational hypothesis to the language of maths
- Deals with specific values (or ranges of values) of population parameters
  - Mean of a given population can be hypothesised do be of a given value
  - We can hypothesise a difference in means between two populations

Statistical hypothesis

- *Conceptual hypothesis:* Climbers have relatively longer arms than non-climbers
- *Operational hypothesis:* Elite climbers have, on average, a higher ape index than general population
- *Statistical hypothesis:* $\mu_{\text{AI\_climb}} > \mu_{\text{AI\_gen}}$

Remember

- We are interested in *population parameters*
- However, we cannot measure them
- We can *estimate* them based on *sample statistics*

Testing hypotheses

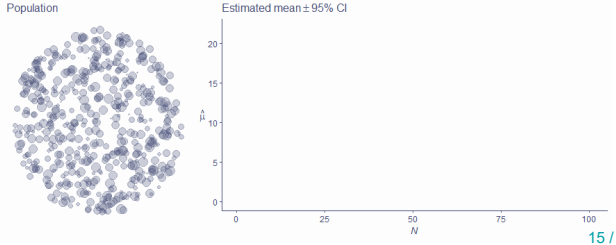
- So we measure a climber and a non-climber and compare them to test our hypothesis
- We find that the climber has a higher AI than the non-climber
- Hypothesis confirmed; we happy

We happy?

No, the individuals might not be representative of the populations

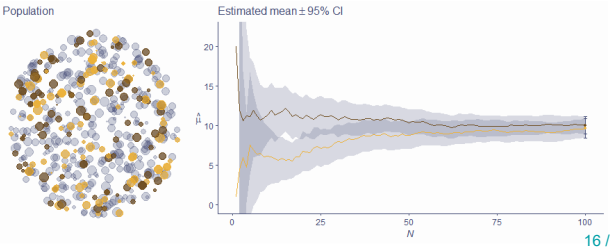
Problem with samples

- We need to collect a larger sample
- However, the principled problem remains: sample mean might not reflect μ accurately



The bigger, the better!

- There are statistical fluctuations; they get less important as *N* get bigger
- Means converge to the true value of μ as *N* increases
- CIs get exponentially smaller with *N*; statistical power increases
- False positives (and negatives!) happen



Decisions, decisions

- How do we decide that a difference/effect in our sample actually exists in population?
- One possible way is using **Null Hypothesis Significance Testing** (NHST)
  - There is strong criticism of this approach

    It is, nonetheless, very widely used
  -
  - Alternatives exist!

NHST

1. Formulate a research hypothesis (from conceptual to statistical)
2. Formulate the null hypothesis
3. Choose appropriate test statistic
4. Define the probability distribution of the test statistic under the null hypothesis
5. Gather and analyse (*enough*) data: calculate sample test statistic
6. Get the probability of the value you got under the null hypothesis
7. If the observed value is *likely under the null*, **retain the null**
8. If it is *unlikely under the null*, **reject the null** in favour of research hypothesis, celebrate!

Hypotheses

- Back to climbers and ape index
- Rather than a directional hypotheses (climbers have longer arms than non-climbers), it's more useful to formulate a hypothesis of *some* difference or effect
- *Statistical hypothesis:* $\mu_{\text{AI\_climb}} \neq \mu_{\text{AI\_gen}}$

The null hypothesis

- Negation of the statistical hypothesis
- Very often about no difference/effect (but not necessarily)
- *Statistical (alternative) hypothesis:* $H_1 : \mu_{\text{AI\_climb}} \neq \mu_{\text{AI\_gen}}$
- *Null hypothesis:* $H_0 : \mu_{\text{AI\_climb}} = \mu_{\text{AI\_gen}}$
- H_1 and H_0 represent *alternative realities* (like parallel universes!)
  - One where there is a difference of effect
  - One where there isn't one

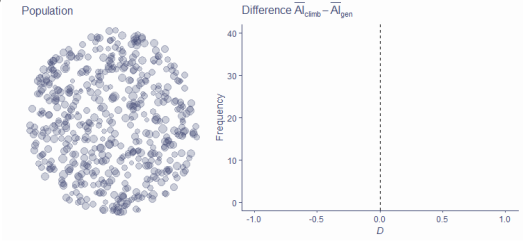
NHST is about deciding which one of the two realities we live in

Test statistic

- Mathematical expressions of what we're measuring (difference, effect, relationship...)
- There are many available test statistics, useful for different scenarios
- For now, let's just take simple difference in means:
  $D = \overline{\text{AI}}_{climb} - \overline{\text{AI}}_{gen}$
- **If null hypothesis is true**, we'd expect $D = 0$, *i.e.*, no difference between climbers' and non-climbers' AI

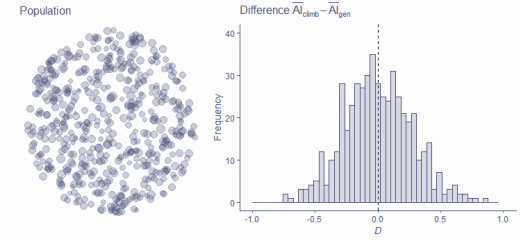
Distribution of test statistic under H_0

- H_0 represents a world where there is *no difference* in average ape index between elite climbers and the general population
- Even if true difference in population (Δ; delta) is zero, *D can be non-zero in sample* (here *N* = 30)
- For simplicity, assume AI_{gen} is normally distributed in population with $\mu = 0$ and $\sigma = 1$



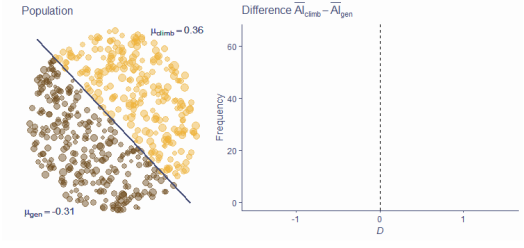
Distribution of test statistic under H_0

- Expected value of D under H_0 is 0
- More often than not D will not be equal to 0 in sample
- Small departures from 0 are common, large ones are rare
- Distribution of test statistic is dependent on N!



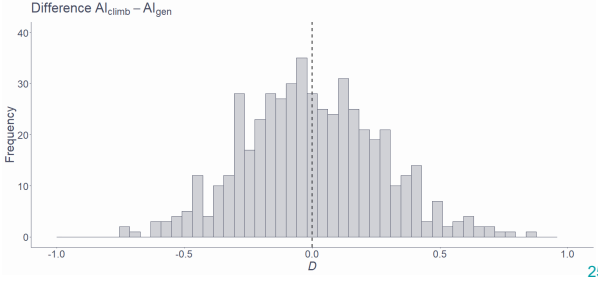
Distribution of test statistic under alternative hypothesis

- H_1 represents a world where there *is a difference* in average ape index between elite climbers and the general population
- If H_1 is true, the sampling distribution of the test statistics is not centred around zero
- Sometimes, a null result can still be observed (false negative; Type II error)



Probability of test statistic under H_0

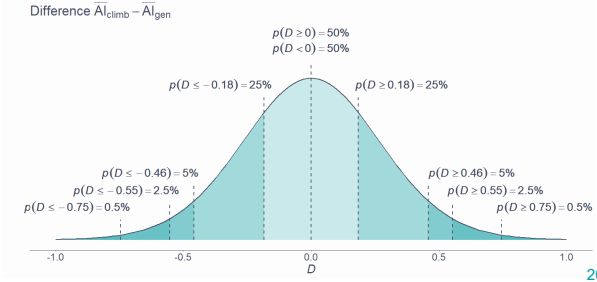
- Once we know what the distribution of our test statistic is, we can assess the probability of getting any given observed value *or a more extreme value* of *D*



Gather data and calculate the test statistic

- Say we collected AI measurements from 30 climbers and 30 non-climbers
- We calculated the mean difference, $D = 0.47$

Calculate probability of observed statistic under H_0



The *p*-value

The *p*-value is the probability of getting a test statistic *at least as extreme* as the one observed *if the null hypothesis is really true*

- Tells us how likely our data are *if there is no difference/effect in population*
- **Does not** tell us the probability of H_0 or H_1 being true
- **Does not** tell us the probability of our data happening "by chance alone"

Decision

- So we have
  - Data
  - Test statistic
  - Distribution of test statistic
  - *p*(test_stat) under H_0
- What now?
- We *reject* H_0 and *accept* H_1 if we judge our result to be unlikely under H_0
- We *retain* H_0 if we judge the result to be likely under it

How likely is likely enough?

- This is an **arbitrary** choice!
- Commonly used *significance levels* are
  - 5% (.05; most common in psychology)
  - 1% (.01)
  - 0.1% (.001)
- If *p*-value is less than our chosen significance level, we call the result *statistically significant* (sufficiently unlikely under H_0)

Significance level must be chosen before results are analysed!

What about the ape index?

- We found a mean difference in AI between climbers and non-climbers of 0.47
- This statistic has an associated *p*-value = .093
- Under the most common significance level in psychology (.05), this is **not a statistically significant** difference
- We thus *retain* the null hypothesis and report not having found a difference: our hypothesis was not supported by the data
- The difference we observed is not big enough for us to dismiss the assumption that we live in the world of H_0

Take-home message

- **Hypotheses** should be clearly formulated, *testable*, and *operationalised*
- **Statistical hypotheses** are statements about values of some parameters
- **Null hypothesis** (usually, parameter is equal to 0) is the one we test (in NHST framework)

We can only observe *samples*, but we are interested in *populations*

- Due to statistical fluctuations, we can find a relationship in sample even if one doesn't exist in population
- **NHST** is one way of deciding if sample result holds in population: understanding it is crucial!

The *p*-value is the probability of getting a test statistic *at least as extreme* as the one observed *if the null hypothesis is really true*